INTERNATIONAL RESEARCH JOURNAL IN ADVANCED ENGINEERING AND TECHNOLOGY (IRJAET) E - ISSN: 2454-4752 P - ISSN : 2454-4744
VOL 2 ISSUE 6 (2016) PAGES 1427 - 1431
RECEIVED : 3/10/2016. PUBLISHED: 05/12/2016
December 5, 2016

EDGE – ODD GRACEFUL LABELINGON CIRCULANT GRAPHS

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Abstract:

Let G = (V,E) be a simple , finite , undirected and connected graph. A graph G = (V,E) with p vertices and q edges is said to have an edge – odd graceful labeling if there exists a bijection f from E to {1,3,5…..2q-1} so that the induced mapping f^+ from V to 0,1,2,….2q-1 given by $f^+(x) = \sum \{ f(xy / xy \in E \}$ (mod │2q│). In this paper, we have constructed an edge – odd graceful labeling on circulant graphs C_n(1,2,3,4) and C_n (1,2,3,4,5) for odd n , n$\in$ I . Here (1,,2,3,4) and (1,2,3,4,5) are the generating sets.

Keywords : Labeling , Graceful labeling , odd – graceful labeling , Edge graceful labeling, Edge – odd graceful labeling , circulant graph.

Mathematics Subject Classification : 05C78.

1. INTRODUCTION

Throughout this paper ,G = (V, E) denotes a simple , finite connected and undirected graph. Let p and q denote the order and size respectively of the graph G . A graph labeling is an assignment of integers to the vertices or edges or both , subject to certain conditions . In 1967 , Rosa introduced a labeling of G called graceful labeling which is an injective function f from V(G) to the set {0,1,2,…..q} such that each edge xy is assigned with the label │f(x) – f(y) │. S.Lo introduced the concept of edge – graceful labeling in 1985. A. Solairaju and K.Chithra introduced a new type of labeling of a graph G with q edges called an edge – odd graceful labeling if there is a bijection f from the edges of the graph to the set {1,3,5…..2q-1} such that each vertex is assigned , the sum of all the edges incident to it (mod 2q) and the resulting vertex labels are distinct .For basic definitions, we can refer [5] and [11].

2. MAIN RESULTS

Edge – odd graceful labeling on circulant graph with generating set (1,2,3,4) are classified by using the following theorem,

Theorem 2.1 For odd $n \geq 11$, the circulant graph $G = C_n(1,2,3,4)$ admits edge – odd graceful labeling. Here (1,2,3,4) are the generators of G .

Proof

Let $G = C_n(1,2,3,4)$ be the 10 – regular circulant graph with $n \geq 11$. Let $V(G) = \{V_i / i = 0,1,\ldots.n-1\}$. Here q = 4n .

We can define the function $f : E(G) \rightarrow \{1,3,5\ldots.2q - 1\}$ by

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$$f(V_i V_{i+1}) = \begin{cases} i + 1 & \text{for } i = 0,2,4\ldots n-1 \\ n + 1 + i & \text{for } i = 1,3,5\ldots n-2 \end{cases}$$

$f(V_i V_{i+2}) = 4n - 1 - 2i$ for $i = 0,1,2\ldots n-1$

$f(V_i V_{i+3}) = 5n + 2i + 8$ for $i = 0,1$

$f(V_i V_{i+3}) = 3n + 2i + 8$ for $i = 2,3,\ldots n-1$

$f(V_i V_{i+4}) = 6n + 1 + 2i$ for $i = 0$

$f(V_i V_{i+4}) = 8n - 2i + 1$ for $i = 1,2,\ldots n-1$

It can be verified that the edge label under the labeling f is a bijection from the set $E(C_n(1,2,3,4))$ onto the set $\{1,3,\ldots 2(4n)-1\}$. For every vertex $v \in V(G)$, the vertex – weight $f^+(v)$ of $C_n(1,2,3,4)$ are defined as follows .

Case i) For i = 0,1,2,3.

a) For i = 0

$\sum_{e \in N(v0)} f(e)$ = 1+ 4n -1 +5n +8+6n+1+(n-1+1)+4n-1-2(n-2) +5n+2(n-3) +8+6n+1+2(n-4)

= 33n + 7

b) For i = 1

$\sum_{e \in N(v1)} f(e)$ = n+1+4n-1-2+5n+2+8+8n-2+1+(0+1) +4n – 1 -2(n-1) +5n +2(n-1) +8+8n – 2(n-3) +1.

= 33n + 21 .

c) For i = 2

$\sum_{e \in N(v2)} f(e)$ = 2+1+4n-1-4+3n+4+8+8n-4+1+n+1+1+4n-1-2(0)+3n+ 2(n-1)+8 +8n-2(n-2) +1.

= 31n + 9.

a) For i = 3

$\sum_{e \in N(v3)} f(e)$ = n +1 +3+4n-1-6+3n+6+8+8n-6+1+n+1+1+4n-1+3n+ 2(n-1)+1.

= 32n + 16 .

Case ii) For i = 4,6,8….n-1.

$\sum_{e\epsilon N(vi)} f(e)$ = i+1+4n-1-2i+3n+2i+8+8n-2i+1+n+1+(i-1)+4n-1-

2(i-2)+3n+2(i-3) +8+8n -2(i-4) +1 .

= 31n – 2i +23.

Case iii) For i = 5,7,…n-2

$\sum_{e\epsilon N(vi)} f(e)$ = n + 1+i+4n-1-2i+3n+2i+8+8n-2i+1+i+1+4n-1-2(i-2)+3n+

2(i-3)+8+8n-2(i-4) +1 .

= 31 n -2i +23 .

Since the generating set contains four elements (1,2,3,4), by taking modulo 4n for the integers , we have the vertex – weights induced mapping f^+ from V(G) to 0,1,2,……4n-1 .

III Edge – odd graceful labeling on circulant graph with generating set (1,2,3,4,5)

Theorem 3.1 For odd $n \geq 11$, the circulant graph $G = C_n(1,2,3,4,5)$ admits edge – odd graceful labeling. Here (1,2,3,4,5) are the generators of G .

Proof

Let $G = C_n (1,2,3,4,5)$ be the 10 – regular circulant graph with $n \geq 11$.

Let $V(G) = \{V_i / i = 0,1,….n-1\}$.

Here q = 5n .Define the function $f : E(G) \rightarrow \{1,3,5….2q – 1\}$ by

$$f(V_i V_{i+1}) = \begin{cases} i + 1 & \text{for } i = 0,2,4….n - 1 \\ n + 1 + i & \text{for } i = 1,3,5….n-2 \end{cases}$$

$f(V_i V_{i+2}) = 4n – 1 – 2i$ for $i = 0,1,2…n- 1$

$f(V_i V_{i+3}) = 5n + 2i + 8$ for $i = 0,1$

$f(V_i V_{i+3}) = 3n + 2i + 8$ for $i = 2,3,….n - 1$

$f(V_i V_{i+4}) = 6n + 1 + 2i$ for $i = 0$

$f(V_i V_{i+4}) = 8n – 2i + 1$ for $i = 1,2 ,…..n- 1$

$f(V_i V_{i+5}) = 9n +i +10$ for $i = 0$

$f(V_i V_{i+5}) = 8n + 2i -1$ for $i = 1,2,….n – 1$.

It can be verified that the edge label under the labeling f is a bijection from the set

$E(C_n(1,2,3,4))$ onto the set {1,3,…..2(4n) - 1}.

For every vertex v ϵ V (G) , the vertex – weight f^+ (v) of C_n (1,2,3,4) are defined as follows .

Case i) For i = 0,1,2,3,4

a) *For i = 0*

$\sum_{e\epsilon N(v0)} f(e)$ = 1+ 4n -1 +5n +8+6n+1+9n+10+(n-1+1)+4n-1-2(n-2) +5n+

2(n-3) +8+6n+1+ 2(n-4)+ 9n +(n-5)+10.

= 52n + 22.

b) *For i = 1*

$\sum_{e\epsilon N(v1)} f(e)$ = n+1+1+4n-1-2+5n+2+8+8n-2+1+ 8n+2-1+(0+1) +4n –

1-2(n-1) +5n + 2(n-2) +8+6n+1+2(n-3)+9n+(n-4)+10 .

= 53n + 17 .

c) *For i = 2*

$\sum_{e\epsilon N(v2)} f(e)$ = 2+1+4n-1-4+3n+4+8+8n-4+1+8n+4-1+n+1+1+4n-1

- 2(0)+3n+2(n-1)+8+8n-2(n-2)+1+8n+2(n-3)-1 .

= 49n + 15.

d) *For i = 3*

$\sum_{e\epsilon N(v3)} f(e)$ = n +1 +3+4n-1-6+3n+6+8+8n-6+1+8n+6-1+n+2+1+4n-1

-2(n+1)+3n+2(0) +8+8n-2(n-1)+1+8n+2(n-2)-1 .

= 46n + 17 .

a) *For i = 4*

$\sum_{e\epsilon N(v4)} f(e)$ = 4+1+4n-1-8+3n+8+8+8n-8+1+8n+8-1+n+3+1+4n-1

-2(n+2)+3n+2(n+1) +8+8n-2(0)+1+8n+2(n-1)-1 .

= 49n+19 .

Case ii) For i = 6,8….n-1.

$\sum_{e\epsilon N(vi)} f(e)$ = i+1+4n-1-2i+3n+2i+8+8n-2i+1+8n+2i-1+n+1+i-1+4n-1-1

-2(i-2)+3n+2(i-3) +8+8n-2(i-4)+1+8n+2(i-5)-1 .

= 47n +2i +3.

Case iii) For i = 7,9,…n-2

$\sum_{e\epsilon N(vi)} f(e)$ = n + 1+i+4n-1-2i+3n+2i+8+8n-2i+1+8n+2i-1+(i-1+1)+4n-1

-2(i-2)+3n+2(i-3) +8+8n-2(i-4)+1+8n+2(i-5) - 1 .

= 47n +2i +3 .

Since the generating set contains five elements (1,2,3,4,5), by taking modulo 5n for the integers , we have the vertex – weights induced mapping f^+ from V(G) to 0,1,2,……5n-1 .

CONCLUSION

In this paper, we obtained Edge – odd graceful labeling on circulant graphs with generating sets (1,2,3,4) and (1,2,3,4,5) . In future, we propose to extend the study for Circulant graphs with higher order generating sets.

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