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TIME VARYING SYSTEM'S STABILITY

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ABSTRACT

In this paper we consider a non linear time-varying discrete-time system, a notion of poles and zeros are developed in terms of factorizations of operator polynomials with time-varying coefficients. In the discrete-time case, it is shown that factorizations are computed by solving a nonlinear recursive difference equation with time-varying coefficients.

Key words – Difference equation, Time varying system, Zeros and Poles, Stability.

AMS Subject Classification: 39 A10, 39A11, 39A20

1. INTRODUCTION

Difference equations are the appropriate mathematical representation for discrete processes, which have special importance in areas such as Signals and systems, filter design, Noise reduction techniques and so on[5,6]. Difference equations of order greater than one are much less studied, and they have great importance in applications where the state (for example, the size of a population) after n steps depends on the previous k +1 states [3].

The basic idea here is to consider systems with changes occurring discretely. We can't really observe such organisms continuously, so we just monitor the quantities of interest at discrete time intervals[2]. An example would be locations of individuals (which move continuously, but we only observe at discrete time intervals) [4]. This is the basic idea of time series analysis, which are statistical approaches to describing, predicting and controlling the behavior of a time-dependent system [1].

The paper is organized as follows. In Section 2, solutions of the higher order input / output difference equation and its behavior are analyzed through poles and zeros in a discrete time interval. Cascade realization of the difference equation which involves the delay is also modeled and extends the stability analysis of such models i.e filters. Finally some examples are illustrated to enrich the results. Section 4 concludes the paper.

2. MAIN RESULTS

Consider the linear discrete-time system of second-order input/output difference equation

$$y(k+2) + a_1(k)y(k+1) + a_0(k)y(k) = b(k)u(k) \tag{1}$$

Where $y(k)$ is the output at time k, $u(k)$ is the input at time k and $a_0(k), a_1(k)$ are elements of A (the set of all functions defined on Z). For any positive integer, let z^i denote the i-step left shift operator on A which is defined as

$$z^i f(k) = f(k+i), f \in A.$$

For any $a(k) \in A$, let $a(k)z^i$ denote the operator on A defined by

$$a(k)z^i f(k) = a(k)f(k+i).$$

Then we can write (1) in the operator form

$$\left(z^2 + a_1(k)z + a_0(k)\right)y(k) = b(k)u(k) \quad (2)$$

Suppose that there exist functions $p_1(k), p_2(k) \in A$ such that

$$\left(z^2 + a_1(k)z + a_0(k)\right)y(k) = \left(z - p_1(k)\right)\left[\left(z - p_2(k)\right)y(k)\right] \quad (3)$$

It follows from (3) that the given system can be viewed as a cascade connection of two first-order subsystems.

$$v(k) = \left(z - p_2(k)\right)y(k) \quad (4)$$

So that

$$y(k+1) - p_2(k)y(k) = v(k) \quad (5)$$

Inserting the expression (4) for $v(k)$ into (3) and using (2), we have that

$$\left(z - p_1(k)\right)v(k) = b(k)u(k)$$

or

$$v(k+1) - p_1(k)v(k) = b(k)u(k) \quad (6)$$

From (5) and (6) , we have the cascade realization of the system shown in Figure 1 below.

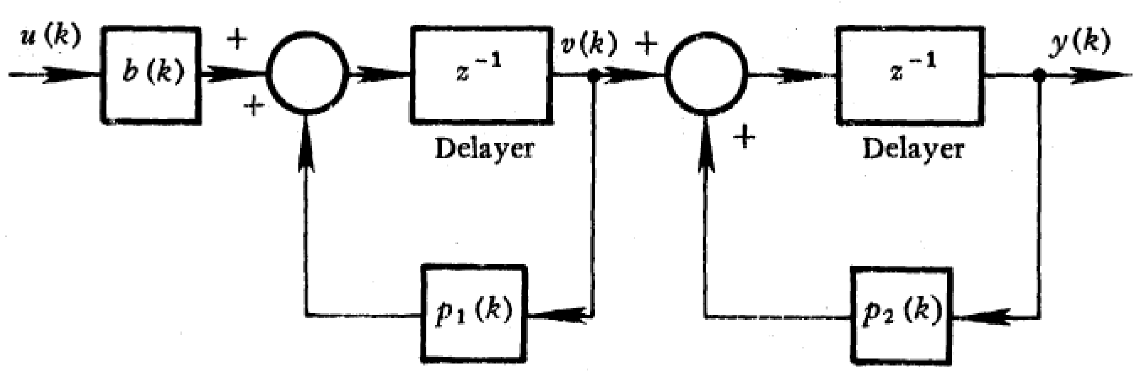

Figure 1 Cascade Realization of System

Initially characterize the cascade decomposition in terms of a polynomial factorization, and then consider the existence and construction of factorizations. In the above Cascade realization system, the delay block is used to provide the previous output.

Again suppose that there exist $p_1(k), p_2(k) \in A$ such that (3) is satisfied. Now define a product

$$\left(z - p_1(k)\right) \circ \left(z - p_2(k)\right)$$

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and hence

$$\left[\left(z-p_1(k)\right)\circ\left(z-p_2(k)\right)\right]y(k)=\left(z-p_1(k)\right)\left[\left(z-p_2(k)\right)y(k)\right] \quad (7)$$

Expanding the right side of (7) , and simplifying we get

$$(z-p_1(k)\circ\left(z-p_2(k)\right)=z^2-\left(p_1(k)+p_2(k+1)\right)z+p_1(k)+p_2(k) \quad (8)$$

Equation (8) shows that the multiplication ∘ is the usual polynomial multiplication except that

$$z\circ p_2(k)=p_2(k+1)z \quad (9)$$

We now consider the existence of a skew polynomial factorization of the form (8). Combining (3), (7), and (8)

$$z^2-\left(p_1(k)+p_2(k+1)\right)z+p_1(k)\,p_2(k)=z^2+a_1(k)z+a_0(k) \quad (10)$$

Equating coefficients of (2) in (10), we see that there is a factorization of the form (8) if and only if

$$p_1(k)+p_2(k+1)=-a_1(k) \quad (11)$$

$$p_1(k)p_2(k)=a_0(k) \quad (12)$$

Multiplying both sides of (11) by $p_2(k)$ and using (12), we obtain

$$p_2(k+1)p_2(k)+a_1(k\,)p_2(k)+a_0(k\,)=0 \quad (13)$$

Note that (13) is a nonlinear first-order difference equation with time-varying coefficients. It is also interesting to note that the left side of (13) looks like the polynomial $z^2+a_1(k)+a_0(k)$ evaluated at $p_2(k)=z$ with the constraints

$$z=p_2(K)\text{ and } z^2=\ p_2(K+1)p_2(K)$$

Given the initial value $p_2(k_0)$ at initial time k_0, we can compute $p_2(k)$ for $k>k_0$. Now by solving (11) and (12) recursively. If $p_2(k)\neq 0$ for all $k\geq k_0$, then the solution is unique. If $p_2(k_0)$ selected at random, the probability is zero that $p_2(k)$ will be zero for some value of $k\geq k_0$. In other words, for almost all initial values $P_2(k_0)$ has a unique solution $p_2(k)$ with $p_2(k)\neq 0$ for all $k\geq k_0$, furthermore, $p_1(k)$ is computed from (11).

CONCLUSION

The theory is applied to the study of the input /output response system's stability such as filter. It is also shown that if a time-varying analogue of the difference equation is invertible, the zero-input response are decomposed and associated with the poles. Finally Cascade realization and direct form of difference equations are modeled.

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